MATH 512, SPRING 2022
HOMEWORK 3, DUE FRIDAY APRIL 22

Problem 1. *Suppose that $\mathbb{P}$ is a forcing notion that does not preserve stationary subsets of ω_1. Show that there is a collection $\langle D_\alpha \mid \alpha < \omega_1 \rangle$ of dense subsets of $\mathbb{P}$, such that there is no filter $G \subset \mathbb{P}$ in the ground model that meets them all.*

Remark 1. The above problem shows that we cannot have a forcing axiom for posets that are not stationary set preserving for subsets of ω_1.

Recall that $\mathbb{P}$ is proper if it preserves stationary subsets of $[\lambda]^\omega$ for all uncountable cardinals λ.

Problem 2. *Show that any c.c.c forcing is proper.*

Problem 3. *Show that any countably closed forcing is proper.*

Problem 4. *Show that if $\mathbb{P}$ is proper, then $\mathbb{P}$ preserves stationary subsets of ω_1.*

Recall that q is $(M, \mathbb{P})$-generic if for every maximal antichain $A \in M$, $A \cap M$ is predense below q (i.e. every $r \leq q$ is compatible with some $s \in A \cap M$).

Problem 5. *Suppose that $\mathbb{P}$ is a forcing notion and $q \in \mathbb{P}$. Let λ be large enough and $M \prec H_\lambda$ be such that $\mathbb{P} \in M$. Show that the following are equivalent:*

1. *q is $(M, \mathbb{P})$-generic.*
2. *$q \Vdash$ "$\dot{G} \cap M$ is a $\mathbb{P}$-generic filter over M".*